"Homework should never be stopped, but they could, however, limit the amount of work they bring home" (E.H-D.).
"Overall, I think that reading this article has changed my perspective on my view of homework. Although I already do my math homework, and the Growing Success document states, 'assignments for evaluation must not include ongoing homework that students do to consolidate their knowledge,' I will continue to do my homework and can now see the purpose of doing so" (J.J.).
My Grade 11 Workplace class could not see the need for any homework. The students felt that as long as they were able to summarize their notes each day, they would not have to spend any time at home working on concepts. They also felt that since the pace of their course enabled them to do a single concept in three or four days, and since the tests were cumulative, they did not have to study or do homework in order to succeed in the course.

From a grassroots perspective, I suggest that the Gazette can be a tool in your classroom for conversationsjust as it was with Todd's column, in my classroom. In this case, the column was inspired by an NCTM position. The NCTM considers many such topics that are relevant and meaningful at all levels of education.

## NCTM important dates to note:

NCTM Annual Conference, April 25-28, 2018
Walter E. Washington Convention Center, Washington, DC.

## NCTM top three things of interest on the website right

 now:1) At a classroom level - Under the Classroom Resources tab, there is a section entitled "Activities with Rigor and Coherence - ARCs." There are K-12 resources that get at the heart of understanding.
2) At a provincial level - Every Student Succeeds Act ESSA Toolkit is located on the website and is very helpful to teachers in their preparation of lessons and activities.
3) At a university level - January 1, 1970 was issue 1 of the Journal for Research in Mathematics Education. Each year, many topics with regard to education are explored and prepared in a meaningful wayeverything from how what is taught affects students to what should be selected to teach children.

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# A Math JAMS: Students Analyzing, Comparing, And Building on One Another's Work 



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Tina Rapke is an assistant professor at York University in the Faculty of Education. She views her research and teaching as related, seamless, and complementary. She and Naomi have been having a lot of fun Jamming!

Math Jams is a fun, engaging, active way for students to employ deep approaches to learning mathematics at any level, from elementary school to university. The creation of Math Jams was inspired by Comics Jams and emerging mathematics education research that focuses on effective use of student thinking in the classrooms (e.g., NCTM, 2014; Smith, Hughes, Engle, \& Stein, 2009). Here we detail exactly what Math Jams are, talk about implementation, and consider why you would want to use Math Jams in your own classroom. We also give you some sample Math Jams from a Grade 4 classroom and a university education classroom. We hope that after reading this article, you will be inspired to use Math Jams in your own classroom and encourage your friends and colleagues to try out Math Jams.

## What are Math Jams?

Math Jams are a spin-off of Comics Jams. Let us first explain Comics Jams. Comics Jams are attributed to Rupert Bottenberg, a Montreal-based illustrator, who established "COMIX JAMS" as a collaborative medium for illustrators and the general public to create comics together. We were
drawn to the collaborative nature of this medium, mindful of the educational uses of comics in the classroom (Bitz, 2009). In a Comics Jam, each member of a group-usually three or four people-starts one story by illustrating a single frame (both picture and words). Each subsequent member continues the storyline by first analyzing what has already been contributed, and then building on the initial story by completing the next comic frame. This continues until all group members have contributed one frame to each of the stories in the group. Each comic is then returned to the group member who started that story, and the comics are then discussed by the group. Group members gain insights into how the others took up their ideas. We have been using Comics Jams to engage students in weekly readings and lectures. Students craft collective comics on a given topic related to course themes by responding to a prompt such as, "What are you left wondering about?" or, "Create a comic that addresses [a particular theme or idea]."

We have adapted Comics Jams to fit within mathematics classrooms. For a Math Jam, students are also placed in groups of three or four members, but they work together as a group on a single given mathematics task on paper or whiteboards situated around the classroom. We monitor the mathematical ideas that students are negotiating and recording. Once we see that each group has something that another group can analyze and/or compare to their own strategies and build upon, we tell the students, "It's time to Jam!," meaning that each group needs to move to the workspace of another group. We tell students to first try as a group to make sense of what the other group left them and then decide on at least one idea or at least 20 percent of the work to keep and build upon to produce a revised response that clearly demonstrates the mathematical thinking that took place during the solving process. Note that we have only had students move once and only build on one other group's ideas. At the end, students return to their original workspace to reflect on how their classmates have built on their mathematical ideas.

## Why should you try out Math Jams in your own classroom?

You should try out Math Jams in your practice because they: 1) are engaging and active ways for students to employ deep approaches to learning mathematics, and 2) can be used to respond to calls in the research about emphasizing students' mathematical thinking in classrooms.

The benefits of Math Jams can be connected to several areas of research, including student engagement, deep approaches to learning, and active learning. Some researchers (e.g., Fredricks, Blumenfeld, \& Paris, 2004) discuss student engagement in terms of behavioural
engagement and emotional engagement, using descriptors like enjoyment and interest (e.g., Trowler, 2010). When we have used Math Jams, we have observed students laughing and smiling. In one university classroom, dancing actually broke out during a Math Jam. This is significant, as students' enjoyment of mathematics is directly related to their achievement in mathematics (García, Rodriguez, Betts, Areces, \& Gonzalez-Castro, 2016). Math Jams can also be spoken of in terms of Lambert and Sugita's (2016) conceptualization of engagement "not simply as on-task behaviour, but as engagement in the practices that define mathematical activity" (p. 348). They go on to speak about students engaging with the mathematical strategies of other students, which is also indicative of deep approaches to learning. Seeing things from different perspectives is one way to describe deep approaches to learning (Marton, Beaty, \& Dall'Alba, 1993). Math Jams are designed so that students have to make sense of others' solution strategies (see things from a different perspective), and build on them. Math Jams also indicate active learning, as they encourage students to take on an active role in the learning process. In this way, Math Jams might be considered antithetical to the traditional lecture, wherein students sit passively, take notes, and formulate pertinent questions to ask (Röj-Lindberg, 2001).

The mathematics education community clearly values students' mathematical thinking in the classroom. This is evident in Ontario mathematics curricula, where thinking is one of the categories in the achievement chart, and where there are questions addressing thinking in Ontario's provincial testing (EQAO). However, the actual implementation of using students' mathematical thinking in classrooms has proven to be challenging and complex (Leatham, Peterson, Stockero, \& Van Zoest, 2015). It is suggested that "[e]ffective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014), but there are few practical, explicit, and easily implemented strategies.

Smith and colleagues (2009) provide one of the few exceptions of explicit suggestions for orchestrating productive mathematical discussions. They offer five practices to help teachers use students' mathematical thinking. In the practice of sequencing (one of the five practices), they clearly articulate, "...the goal is to have student presentations build on each other to develop powerful mathematical ideas" (Smith et al., 2009, p. 554). This process is meant to support students in building shared understanding. This is starkly different from students working individually, with potentially only a few students in the
classroom understanding the topic/strategy. However, the focus is on students' presentations and thus on oral communication.

Math Jams have a similar goal of students building on each other's ideas to develop and spread understanding among students; however, Math Jams are not focused on oral presentation, but rather on written forms. Specifically, Math Jams make it essential that students interact with and analyze each other's written ideas, developing their understandings by ultimately returning to reflect on how their classmates built on their ideas. In fact, NCTM's suggestions about effective teaching, the goal of Smith and colleagues, and our own work with Comics Jams, motivated us to create Math Jams as an explicit strategy that has students analyze and build on one another's mathematical ideas in writing.

## Implementing Math Jams

When implementing Math Jams, it is helpful to think about the following: the types of problems to use, directions to give when launching, timing, and how to support students to engage with their classmates' work. We suggest using problems with several entry points, as these will allow for several explanations/ideas to be built upon, clarified, and perhaps related to each other. Furthermore, if students do not initially understand the problem, there are many questions the teacher can ask before and after the Jam. (We will highlight such questions in the examples we provide.) Having problems with multiple entry points and an attentive teacher who is closely monitoring ideas as they emerge can increase the opportunities for students to communicate and engage with one another's ideas. In terms of timing, we were concerned that if the jam occurred too soon after launching the task, then students would have difficulties finding an idea to build upon, and if too late, then students would have too little to do.

We recommend providing explicit instructions when launching Math Jams, as these can help address concerns about students' desire for ownership over their individual ideas (e.g., Bruce, 2007). Indeed, in our implementation of Math Jams at the elementary level, students mentioned "stealing ideas," even after the teacher was very purposeful in describing that Math Jams required students to share and build on each other's ideas, highlighting the benefits of doing so. For example, our collaborating elementary teacher explained to students that the "basic idea of a Math Jam is about sharing ideas. But not only sharing ideas by talking about them, we're actually going to be sharing ideas by trading them." He reiterated that: students were to work together and not alone; when they engaged another group's work, they would decide what part of that group's work they would carry forward to work up with their own ideas; and
finally, at least one idea from the other group's work had to be built upon. The teacher then asked the students why they might be doing a Math Jam and asked what the students thought would be the benefits of doing a Math Jam. One student said, "We could get, like, different ideas... we'll learn from them," and another student said that he could steal ideas. The teacher was very particular in clarifying that "we're not stealing because we know we're passing them around and we're accepting them and giving them freely." We have implemented Math Jams in classrooms where teachers have an awareness of collective understanding (as described in Martin, Towers, \& Pirie, 2006), which may have played a role in their success. We have experienced Math Jams in terms of students sharing responsibility with their group mates, with ideas owned by the group collectively, freely shared and accepted.

When supporting students to engage with one another's work, the concepts of sharing and building upon are important when considering how to alleviate students' anxiety about classmates seeing their work. In the literature on peer feedback, there is discussion about alleviating the anxiety students may have with classmates seeing their work by providing positive feedback first (Topping, 1998). In Math Jams, it is anticipated that anxiety may be alleviated by building upon and sharing ideas. When students receive/accept classmates' ideas, the students have already worked on the mathematical problem and are essentially making sense of the ideas in light of their previous engagement with the problem. This means that at first glance, it may appear that students are simply correcting their classmates' work or judging that their ideas are not valuable. However, students are in fact mapping their previous thinking onto the ideas they receive. It may be that a student/group experienced a similar misconception to one present in the ideas they received, and may have chosen to build on the misconception by providing a rationale to explain why it is a misconception. In the language of effective peer feedback (Lu \& Law, 2012), students are identifying growing misconceptions and addressing them with concrete ideas about how to move the shared ideas forward.

## Example of a Math Jam in a Grade 4 Classroom

Our first example of a Math Jam comes from an inner city Grade 4 class that consisted of students with diverse experiences and ways of knowing. The school acknowledges that many of their students struggle with poverty and language barriers. The task given addressed the Grade 4 Ontario mathematics curriculum expectation on proportional relationship, wherein students must
"demonstrate an understanding of simple multiplicative relationships" (Ontario Ministry of Education [OME], 2005, p. 68). Students were told that 36 cookies cost $\$ 12$, and that Hanna pays $\$ 2$, Joanna pays $\$ 3$, Leanna pays $\$ 4$, and Nanna pays $\$ 3$. They were then asked, "How should the cookies be shared fairly based on how much each paid?"

Students worked in groups of two or three on the task. When students had difficulties in how to attempt the problem, teachers asked them about fairness. They asked questions like: "Would it be fair if Joanna and Nanna had a different number of cookies?" "Would it be fair if Hanna had 2 cookies and Leanna had 2 cookies?" "If Hanna had 3 cookies, how many cookies would Leanna have for it to be fair?" These questions demonstrate the multiple entry points the task has. Figure 1 is an example of students' work. The original group wrote in blue and the next group in green.

In the upper left-hand corner (written in blue) is the idea that students decided to keep to build upon from the work that they received. Clearly the students (who received this work) analyzed this work and compared it with their own. This is evidenced as the students analyzed their classmates' work and wrote, "They were doubling the numbers." They then compared it to their own strategies, as is evidenced by them saying, "I used the doubling," meaning that they did not judge the ideas to be invaluable, but actually found them similar to their own initial engagement with the problem. Students are communicating that their classmates' strategy cannot be used to obtain a solution because $4+6+8+6=24$, and in order to get 36 (the total number of cookies), they would have to add 12 more (i.e., 24+12=36). The students then offer a correct answer and check their solution (as they also did for their classmates' guess) by adding up the numbers and confirming that they come to 36 cookies. Essentially, students are demonstrating the Grade 4 mathematical process of reflection by "comparing and adjusting strategies used, by explaining why they think their results are reasonable" (OME, 2005, p. 65).


Figure 1: Example of a Grade 4 Math Jam

## Example of a Math Jam in a University Classroom

Our second example comes from a mathematics education course taken by future teachers. This problem was initially developed to address the Grade 6 curriculum expectation: "solve problems requiring conversion from larger to smaller metric units" (OME, 2005, p. 90). Students were told that a group of eight students are on a field trip when their bus breaks down 40 km away from their school. A teacher takes four of them back to school in her car, travelling at an average speed of $120 \mathrm{~km} / \mathrm{h}$. The other four students start walking toward the school at a steady rate of $6 \mathrm{~km} / \mathrm{h}$. The teacher drops the four at school and then immediately turns around and comes back for the others. Students are asked, "When does the teacher reach the second group of students?" They are given five choices ( 20 mins, 30 mins, 30.5 mins, 38 mins, 39 mins) and asked to indicate and explain their choice. Student worked in groups of three or four.


Figure 2: Example 1 of a Math Jam with future teachers
When students were experiencing difficulties, the instructor asked questions such as: "How long does it take the teacher to travel 60 km ?" "How long does it take the teacher to travel 30 km ?" "How long does it take the teacher to travel 40 km ?" These questions demonstrate the multiple entry points students can take when approaching the problem. The black writing in Figure 2 is from the original group, and the second group wrote in green. Notice in the upper right-hand corner of Figure 2, students have used division and the numbers 120 and 40 to obtain an answer of "30 mins." The second group of students indicate that they analyzed their classmates' work by circling "mins" and writing "hours?" In the bottom left-hand corner, they address/build on ideas about division and the use of the numbers 120 and 40 by offering a different strategy and obtaining a solution of " $1 / 3 \mathrm{hr}=20$ mins."

In Figure 3, students have indicated that they have analyzed their classmates' work (the black writing) by extrapolating/explaining some of the ideas. Specifically, in the upper left-hand corner of Figure 3, " 38 km " is written in black. Underneath the diagram, they have explained where the 38 km comes from. The students first compute how far the students have walked in 20 minutes (the time for the teacher to drive back to the school), and then explain that the teacher would have to travel back at most 38 km .

## Discussion

We note similarities between the Grade 4 class and the teacher education class. Notice that in the Math Jams we have provided, students in both classes have improved each other's work, making the thinking more clear and concise (e.g., Figure 3), or they have addressed misconceptions (e.g., Figures 1 and 2). With regard to timing, when students seem to be working faster, or when the Jam occurs later in the lesson, students will polish each other's work, adding or deleting text to make the thinking more clear and concise. When the Jam occurs soon after the launch of the task, or if students are working more slowly, students tend to identify misconceptions and offer a rationale and a better idea. It seems that, regardless of the timing of the Jam or of the level of learning, students still analyze and build upon one another's work.


Figure 3: Example 2 of a Math Jam with future teachers

In Figure 3, students have built on their classmates' ideas by providing annotations to the original work. They have clarified their classmates' ideas, as they have provided details as to why the original picture contains 38 km. Figures 1 and 2 document Math Jams where students address misconceptions, showing that they have analyzed their classmates' work by indicating that their classmates' solutions/strategies are incorrect. "Building upon" has occurred in the sense of offering a rationale for why the initial group's work is not a solution (e.g., the total is not 36 cookies), or offering a strategy that clarifies that 3 is related to $1 / 3$ of an hour, which means it takes the car $1 / 3$ of an hour, or 20 minutes (not 30 minutes), to travel 40 km at a speed of $120 \mathrm{~km} / \mathrm{h}$. In other words, students have made sense of someone else's solution/perspective in terms of their own thinking by offering a new strategy that involves all the same numbers. Furthermore, Math Jams support students to make sense of why their work was incorrect and address their misconceptions by returning to their original workspace to reflect upon how their classmates have analyzed, compared, and built upon their work. This is in line with research findings that students engaging with classmates' mathematical ideas is indicative of their achievement (Ing et al., 2015).

## Conclusion

We hope that Math Jams can offer you, the reader, a valuable strategy to increase communication and opportunities for your students to interact with one another's ideas. The evidence we have provided here indicates that students in Math Jams are engaging with each other's ideas by analyzing, comparing, and building on those ideas. In Math Jams, mathematics learning is deep, engaged, and active, with students sharing the responsibility of building mathematical solutions and understanding collectively.

It is notable that as students are taking responsibility and acting collectively as a group, the process is not teachercentred, and thus can be noisy. A noisy classroom is often understood by teacher candidates to be evidence that the teacher lacks classroom-management skills. Learning classroom-management skills that address the noise associated with groups is often a priority for beginning teachers (Cakmak, 2008; Pereira \& Gates, 2013). In comparison to other group problem-solving environments that we have experienced, during Math Jams, the room is usually very loud, and it is difficult for the instructor to have students pay attention when the instructor wants to point something out, wrap up the Jam, or have students return to their original workspace. In fact, we experienced this when we were doing Jams at a conference with university academics! The instructor needs to be comfortable with this
type of environment. Having teacher candidates experience this in a teacher education setting may help them understand that a "noisy classroom" does not equal "lack of classroom-management skills." Obviously, we do not view students being loud and focused on a task as a bad thing, and we hope this might even motivate you further to try out Math Jams.

We are also planning to bring some of the ideas of Math Jams back to our use of Comics Jams and are seeing ways that Math Jams and Comics Jams can enhance and support each other. Specifically, we will be encouraging students to polish previous comic frames to help clarify the storyline. We also think it would be interesting to focus on and bring forth the benefits of Math Jams by having students engage in a Comics Jam after they have engaged in a Math Jam. In these ways, students can experience jams in multiple disciplines, as jamming holds the potential to weave through disciplines. Even though we only report on preliminary work and have collected a limited amount of data, we hope that we have convinced you of the potential of Jams, and that you will also use Jams in your own ways and in your own context to maximize the benefits. We look forward to future large-scale studies and hearing and learning from your implementations of Comics and Math Jams.

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